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THE HYPERSONIC BOUNDARY LAYER ON A WEDGE WITH UNIFORM MASS ADDITION AND VISCOUS INTERACTION†

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Abstract—The effects of uniform surface blowing on the hypersonic boundary layer with viscous interaction are investigated analytically and experimentally. For strong and moderate viscous interaction, the heat transfer on a flat plate and a slender wedge is calculated by use of the local similarity technique to solve the boundary layer equations and the tangent wedge approximation to determine the inviscid pressure. The experiments are conducted at Mach numbers of 16 and 20, at unit Reynolds numbers of 2·3 and $1{\cdot}3 \times 10^5\ \text{ft}^{-1}$, and at the cold wall condition. The analytical and experimental results are in good agreement. For moderate blowing, it is found that the effects of viscous interaction dominate the flow when the interaction is strong and that the effects of blowing become more important as the strength of the viscous interaction decreases.

NOMENCLATURE

- A, defined in equation (18);
- C, $\mu^*/\mu\ T/T^*$ where (*) refers to Eckert reference enthalpy conditions;
- C_H, $Q_w/\rho_\infty u_\infty(H_t - H_w)$;
- C_p, specific heat at constant pressure;
- g, H/H_e or H/H_t;
- H, total enthalpy;
- K, $M_\infty(\theta_b + \mathrm{d}\delta^*/\mathrm{d}x)$;
- K_b, $M_\infty\theta_b$;
- M, Mach number;
- m, $\rho_w v_w/\rho_\infty u_e$, mass injection or blowing parameter;
- n, exponent of variation of pressure with x; determined in equation (16);
- P, $\bar{p}/\bar{p}_\infty$, dimensionless pressure;
- Q_w, heat transfer rate at the wall [W/cm²];
- Re_x, $(\rho_\infty u_\infty x)/\mu_\infty$;
- T, temperature;
- u, tangential velocity component;
- v, normal velocity component;
- x, y, coordinates along and normal to the wall;
- γ, ratio of specific heats;
- δ^*, boundary-layer displacement thickness;
- ε, $(\gamma - 1)/(\gamma + 1)$;
- η, $\frac{u_e}{\sqrt{(2\xi)}}\int_0^\infty \rho\,\mathrm{d}y$, transformed coordinate;
- θ_b, wedge half-angle, angle of attack of flat plate;
- μ, viscosity;
- ν, μ/ρ, kinematic viscosity;
- ξ, $\int_0^x \rho_e\mu_e u_e\,\mathrm{d}x$, transformed coordinate;
- $\bar{\chi}$, $M_\infty^3\sqrt{(C/Re_x)}$, viscous interaction parameter;
- τ, shear stress.

Subscripts

- ∞, conditions in the free stream;
- e, conditions at the edge of the boundary layer;
- w, conditions at the wall;
- t, stagnation conditions.

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1. INTRODUCTION

THE AERODYNAMIC performance of a reentry vehicle can be strongly influenced by surface blowing due to an ablating heat shield. Blowing tends to increase surface pressure and to decrease viscous drag and heat transfer. However, at high altitude the boundary layer on reentry vehicles can become sufficiently thick that it develops a strong interaction with the inviscid flow and thereby induces a large and favorable pressure gradient. The consequences of this viscid–inviscid interaction, which is characterized by a parameter that compares the viscous surface pressure to the inviscid surface pressure, are that the viscous drag and heat transfer are increased. Since on a reentry vehicle surface blowing and hypersonic viscous interaction can occur simultaneously and can produce opposing effects, it is desirable to have analytical means to study a hypersonic boundary layer when both effects are present, but the required theoretical method must be flexible enough that the effects of a general blowing distribution for the entire range of hypersonic interaction (strong and weak) can be considered. Presently, no exact or approximate solutions of this dimension exist.

Existing theories on boundary-layer blowing have primarily been limited to the special case of self-similar flows. For the flat plate, the solution of Emmons and Leigh [1] gives a complete tabulation of the variation of the boundary-layer parameters with the blowing parameter. Further tabulations of self-similar flows with blowing are presented in [2]. Recently, the effects of external pressure gradient [3] and surface temperature [4, 5] on boundary layer flows with blowing were discussed. Exact solutions of nonsimilar cases are few indeed, and the only case that has been studied in detail is flow over a flat plate with uniform blowing [6]. There are numerous studies of hypersonic viscous interaction [7–9]. The combined effects of mass addition and strong hypersonic interaction are discussed in [10, 11]; however, it appears that the combined effects of surface blowing and hypersonic interaction have not been studied for cases in which the magnitude and distribution of the surface blowing is arbitrary and the magnitude of the interaction parameter is arbitrary.

The objective of the present report is to present an analytical and experimental study of the hypersonic boundary layer on a wedge and a flat plate with uniform blowing over the entire range of interaction parameter. In the experimental study, heat transfer to the surface of the plate is measured at flow Mach numbers of 16 and 20, at unit Reynolds numbers of 2·3 and $1{\cdot}3 \times 10^5$ ft^{-1}, and at cold wall conditions. The mass injection rate characterized by the ratio of injected mass flow to the free stream mass flow is varied from 1 to 5 per cent. The theoretical study is conducted by the approximate, local similarity technique. This technique, which usually results in a simple analysis [12], is hampered by the deficiency in tabulated data for self-similar flows with blowing and a pressure gradient. However, simple results that accurately correlate the experimental data are obtained if, in addition to local similarity, a hypersonic boundary layer with specific heat ratio close to unity is assumed.

2. THEORETICAL ANALYSIS

An exact analysis of the hypersonic boundary layer with mass addition and viscous interaction could be obtained in principle by numerically solving the coupled inviscid and viscous equations. But for the present case, it is expedient to employ approximate techniques. There are many approximations that can be used, but the present problem requires that the approximations to the inviscid and viscous flows are simple enough that the interaction between the flows can be studied. The solution to the inviscid equation will be approximated by employment of the strong shock and slender body versions of the tangent wedge approximation. The solution to the boundary-layer equations will be obtained by use of the local similarity technique, the hypersonic approximation, and the numerical solutions of [1].

If a perfect gas, $Pr = 1$, and $\mu \sim T$ are assumed, the two-dimensional equations for a hypersonic boundary layer in a transformed coordinate system are [7]

$$f_{\eta\eta\eta} + ff_{\eta\eta} + \beta(g - f_\eta^2) = 2\xi(f_\eta f_{\eta\xi} - f_\xi f_{\eta\eta}) \quad (1)$$

$$g_{\eta\eta} + fg_\eta = 2\xi(f_\eta g_\xi - f_\xi g_\eta) \quad (2)$$

where

$$\left.\begin{aligned}
\beta &= 2\xi \frac{H_e}{C_P T_e u_e}\frac{\mathrm{d}u_e}{\mathrm{d}\xi} = -\frac{\gamma - 1}{\gamma}\frac{\mathrm{d}\bar{p}_e}{\mathrm{d}x}\frac{1}{\bar{p}_e^2}\int_0^x \bar{p}_e\,\mathrm{d}x \\
\frac{u}{u_e} &= f_\eta; \qquad g = \frac{H}{H_e}; \\
\rho v &= -\rho_e \mu_e u_e \sqrt{(2\xi)}\left(\frac{f}{2\xi} + f_\xi + f_\eta \frac{\partial \eta}{\partial \xi}\right) \\
\eta &= \frac{u_e}{\sqrt{(2\xi)}}\int_0^x \rho\,\mathrm{d}y; \qquad \xi = \int_0^x \rho_e \mu_e u_e\,\mathrm{d}x; \\
P_e &= \frac{\bar{p}_e}{\bar{p}_\infty}.
\end{aligned}\right\} \quad (3)$$

The appropriate boundary conditions are

$$\eta \to \infty: \; f_\eta \to 1; \qquad g \to 1$$

$$\eta = 0: \; f_\eta = 0; \qquad g = g_w;$$

$$f_w = -m\sqrt{\left(\frac{u_e x^2}{2C\nu_\infty}\right)}\left(\int_0^x P_e\,\mathrm{d}x\right)^{-\frac{1}{2}} \quad (4)$$

where

$$m = \rho_w v_w / \rho_\infty u_e.$$

If the external pressure distribution is given, the problem is completely determined by equations (1), (2) and (4). In the present case, the external pressure is to be obtained from a solution of the inviscid equations for flow over an equivalent body whose ordinate equals the displacement thickness plus the local body ordinate. Some of the complexity is eliminated if it is assumed that the local pressure is given by a tangent wedge approximation. When this assumption is made, the full inviscid equations do not have to be considered. Therefore, for hypersonic flow, the pressure is

$$P_e = 1 + \gamma(K)^2\left\{\left[\left(\frac{\gamma+1}{4}\right)^2 + \frac{1}{K^2}\right]^{\frac{1}{2}} + \frac{\gamma+1}{4}\right\} \quad (5)$$

where

$$K = \left(\theta_b + \frac{\mathrm{d}\delta^*}{\mathrm{d}x}\right)M_\infty = K_b + M_\infty\frac{\mathrm{d}\delta^*}{\mathrm{d}x}.$$

If terms of 0 $[(m^2 Re/M_\infty^4)^{\frac{1}{2}}]$ and $0(M_\infty^{-2})$ are neglected, the displacement thickness is

$$\delta^* = \frac{(\gamma - 1)\bar{\chi}}{2P_e M_\infty}\sqrt{\left(2x\int_0^x P_e\,\mathrm{d}x\right)}\int_0^\infty (g - f_\eta^2)\,\mathrm{d}\eta \quad (6)$$

where $\bar{\chi}$ is the flat plate interaction parameter.

It is clear that δ^* is determined from the solution of the boundary-layer equations [equations (1), (2) and (4)] and that these depend on the pressure [equation (5)], which in turn depends on the displacement thickness.

So far the development of the local similarity equations parallels the one presented by Dewey in [12]. However, the present analysis differs from Dewey's in that blowing from a local similarity point of view was not considered in [12].

Equations (1), (2) and (4)–(6) must be solved simultaneously. These equations are coupled nonlinear partial differential equations because the chosen blowing distribution and range of interaction parameter do not permit the self-similar assumption. Therefore, the exact solution of this problem is difficult even with the aid of high-speed computers. However, a simple analytical solution can be obtained by the application of the local similarity method.

Before proceeding to the local similarity solution, it is of interest to discuss the parameters of the problems. The equations show that the parameters are g_w, m, M_∞, θ_b, Re_x and γ; however, these are not necessarily independent. In the case of a flat plate, the parameters are g_w, M_∞, m, γ and $\bar{\chi}$: When $\bar{\chi} \gg 1$, then $P_e \sim \bar{\chi}$, $\delta^*/x \sim (\bar{\chi}/M_\infty^2)^{\frac{1}{2}}$, $f_w \sim m(M_\infty/\sqrt{\bar{\chi}})^3$ and $C_H \sim [(\sqrt{\bar{\chi}})/M_\infty]^3$. In the case of a wedge, the parameters are the same, except that the interaction parameter is $\lambda = \bar{\chi}/K_b^2$: When $\lambda \gg 1$, then $P_e \sim \lambda$, $\delta^*/x\theta_b \sim \lambda^{\frac{1}{2}}$, $f_w \sim mM_\infty^3/(\lambda^{\frac{1}{2}}K_b^2)$ and $C_H \sim (\lambda^{\frac{1}{2}}K_b/M_\infty)^3$.

The trend of the blowing effects on a flow with viscous interaction can be obtained from the above presentation. When m and M_∞ are constant, the effects of blowing decrease as the viscous interaction increases and increase as the interaction decreases. Thus, on a reentry vehicle the relative effect of blowing near the leading edge is less than near the trailing edge. For blowing to have an 0(1) influence on boundary layer, m must be $0(\bar{\chi}^{\frac{1}{2}}/M_\infty^3)$ for a flat plate and $0(\lambda^{\frac{1}{2}}K_b^2/M_\infty^2)$ for a wedge.

In the present local similarity method, f_w is used as the independent variable rather than the usual β. However, the drawback of using f_w as the independent variable is that the resulting solution is exact only when f_w is zero. But it has been shown that the local similarity method predicts heat transfer accurately when the blowing is not too large [13]. When f_w is large, the local similarity method underestimates the heat transfer.

Using f_w as the independent variable, equation (1) becomes

$$f_{\eta\eta\eta} + ff_{\eta\eta} + \beta(f_w)(g - f_\eta^2) = 2\xi\frac{\mathrm{d}f_w}{\mathrm{d}\xi}(f_\eta f_{\eta f_w} - f_{f_w}f_{\eta\eta}) \tag{7}$$

$$\eta \to \infty: \quad f_\eta \to 1; \qquad g \to 1$$

$$\eta = 0: \quad f_\eta = 0; \qquad g = g_w; \qquad f = f_w. \tag{8}$$

Now we shall assume that $2\xi\, \mathrm{d}f_w/\mathrm{d}\xi$ is small such that

$$f\left(f_w, \eta; \beta; 2\xi\frac{\mathrm{d}f_w}{\mathrm{d}\xi}\right) = f_0(\eta; f_w; \beta)\left[1 + 0\left(2\xi\frac{\mathrm{d}f_w}{\mathrm{d}\xi}\right)\right]. \tag{9}$$

In the leading term f_0, the independent variable f_w is a parameter and β is also a parameter that depends on f_w.

Now, if we had a complete tabulation of $f(\eta; f_w; \beta)$ the leading term would be determined and all the boundary-layer parameters evaluated at the local value of f_w. However, we only have the complete tabulation of the flat plate solution $f(\eta; f_w)$. Therefore we will make the hypersonic assumption that

$$\beta = 0\left(\frac{\gamma - 1}{\gamma}\right) \tag{10}$$

and for γ close to unity, the influence of the parameter β can be neglected [8].

The question arises as to why an approximation based on the Catheral *et al.*, solution [6] for uniform blowing was not developed. One can answer this question by referring to [11] for the derivation of the compressible version of the equations used in [6] and noting that an additional pressure gradient term whose order of magnitude is unity and not $(\gamma - 1)/\gamma$ appears in the compressible equations. This additional pressure term, which is a result of the variable viscosity used in the compressible formulation, cannot be neglected by employing the hypersonic boundary-layer assumption, as was done with the β term. Hence, there is no one-to-one correspondence between the incompressible equations of [6] and the compressible equations [11], unless the pressure gradient is identically zero and this is not the case of interest.

Therefore, the leading terms in our local similarity solution are

$$f\left(f_w; \eta; \beta; 2\xi\frac{\mathrm{d}f_w}{\mathrm{d}\xi}\right) \sim f_0(\eta; f_w)$$

$$g \sim (1 - g_w)f_{0\eta} + g_w. \tag{11}$$

The variables f and g can be evaluated from [1] if the local f_w is given. However, f_w depends on the variation of P_e; therefore, the variation in P_e must be determined.

For the local similarity method, let $P_e \sim x^n$. The exponent n is to be evaluated from local values of the boundary-layer parameters. The variation of n is

$$n = \frac{x}{P_e}\frac{dP_e}{dx} \equiv \left(\frac{K^2}{P_e}\frac{dP_e}{dK^2}\right)\left(\frac{x}{K^2}\frac{dK^2}{dx}\right) \qquad (12)$$

where K is the previously defined hypersonic similarity parameter. Since $P_e \sim x^n$, displacement thickness is

$$\frac{\delta^*}{x} = \left(\frac{\gamma-1}{2}\right)\frac{\bar{\chi}}{M_\infty}\left[\sqrt{\frac{2}{(n+1)P_e}}\right] I \qquad (13)$$

where

$$I(f_w; g_w) = \int_0^\infty (g - f_\eta^2)\, d\eta. \qquad (14)$$

It is assumed that I is a slowly varying function with x, and this implies that $\delta^* \sim x^{(1-n)/2}$.

If we restrict the problem to flat surfaces or wedges ($dK_b/dx = 0$) and employ equation (13) and the definition of K, it can be shown that

$$\frac{x}{K^2}\frac{dK^2}{dx} = -\frac{n+1}{K}(K - K_b). \qquad (15)$$

The relationship between n and K is determined by evaluation of dP_e/dK^2 from the tangent wedge formula [equation (5)] and by use of the resulting expression with equation (15) in equation (12). The final equations are

$$n = -\left(1 - \frac{K_b}{K}\right)\frac{K^2}{P_e}\frac{dP_e}{dK^2}\Bigg/\left[1 + \left(\frac{K^2}{P_e}\right)\frac{dP_e}{dK^2}\left(1 - \frac{K_b}{K}\right)\right] \qquad (16)$$

$$\frac{K^2}{P_e}\frac{dP_e}{dK^2} = \left\{A - \gamma\left[\left(\frac{\gamma+1}{4}\right)^2 + \frac{4}{K^2}\right]^{-\frac{1}{2}}\right\}\Bigg/(1 + A) \qquad (17)$$

where

$$A = 1 + \frac{\gamma K^2}{2}\left\{\left[\left(\frac{\gamma+1}{2}\right)^2 + \frac{4}{K^2}\right]^{\frac{1}{2}} + \frac{\gamma+1}{2}\right\}. \qquad (18)$$

If K_b and K are known then n can be calculated, and the pressure distribution is known. For example, $K = K_b$ results in $n = 0$, the zero-interaction result, and $K \gg 1$ results in the strong-interaction self-similar solution result.

The parameter K is related to P_e and the boundary-layer solution by

$$K - K_b = \bar{\chi}\sqrt{\left(\frac{(1-n)^2(\gamma-1)^2}{8(1+n)P_e}\right)}\, I. \qquad (19)$$

Equation (19) relates K, P_e and $1 + n$ to the boundary solution I, which depends on g_w and f_w.

The remaining relationship between the boundary-layer solution and the external pressure is

$$f_w = -\frac{mM_\infty^2}{\bar{\chi}}\sqrt{\left(\frac{1+n}{2P_e}\right)}. \qquad (20)$$

This equation relates f_w to n and P_e.

The procedure for obtaining a solution is as follows. We are given m, $\bar{\chi}$, M_∞^2, g_w and θ_b. A value of K_i is assumed. P_e is calculated from equation (5), and n is calculated from equations (16)–(18). The value of f_w is determined from equation (20). Then $I(f_w, g_w)$ is obtained from the tabulation in [1], and K_{i+1} is calculated from equation (19). If K_{i+1} does not agree with K_i, the iteration is repeated using K_{i+1} until satisfactory convergence is obtained. This iteration procedure can be easily executed with the aid of a desk computer.

If n, P_e and f_w are known, the boundary-layer parameters can be determined from

$$\frac{\tau_w}{\rho_\infty u_e^2} = \frac{Q}{\rho_\infty u_e(H_e - H_w)} = C_H = \sqrt{\left(\frac{P_e(n+1)}{2Re_x}\right)}\, f_{\eta\eta}(0; f_w). \qquad (21)$$

Before concluding this section, it is appropriate to comment on the parameters that characterize the accuracy of the solution. The first parameter is $2\xi(\mathrm{d}\,f_w/\mathrm{d}\xi)$, and the solution is accurate to $0[2\xi(\mathrm{d}\,f_w/\mathrm{d}\xi)]$. This implies, as can be deduced from the equations, that the accuracy increases as the hypersonic interaction increases or as m decreases. The second parameter is β, and the solution is accurate to $0(\beta)$ or to $0[(\gamma - 1)/\gamma]$. Hence, as m increases and $\tau_w/(\rho_\infty u_e^2)$ decreases, the solution is reasonably accurate until $\tau_w/(\rho_\infty u_e^2)$ is $0(\beta_{gw}/m)$ [5]. At this point, further decreases in $\tau_w/(\rho_\infty u_e^2)$ are not allowed unless the effect of β on the solution is considered. Hence, it is inappropriate to discuss the "blow-off" question using the present approximation. Furthermore, it was shown in [4] and [5] that "blow-off" cannot occur when m, β and g_w are all finite.

3. EXPERIMENTS

Heat transfer to a porous flat plate with mass injected through the surface was measured at flow Mach numbers M_∞ of 16 and 20 and unit Reynolds numbers of 2·3 and $1{\cdot}3 \times 10^5$ ft^{-1}. The mass injection rate parameter m was varied from 0·9 to 4·3 per cent, while angle of attack for the surface of the plate ranged from 0 to 16 deg. The experiments were carried out in the Aerospace hypersonic shock tunnel, which is a contoured nozzle facility designed for $M_\infty = 20$ at reservoir conditions of 200 atm and 2000°K. The details of the model, instrumentation and testing techniques are as follows.

The model consists of an aluminum box, the top surface of which is a porous plate. A 30-deg wedge with a 0·003 in. radius of curvature provides the leading edge of the plate, the surface dimensions of which are $8 \times 8\frac{1}{2}$ in. A $\frac{1}{16}$-in. thick stainless steel plate with a 2-μ porosity provides the porous surface of the model. The porous section begins about $\frac{1}{2}$ in. aft of the leading edge. The injected gas, N_2 in this case, is introduced into the plenum and controlled by an external flow metering system.

Flow uniformity of the gas injected from the surface was assured by observation of the flow variations with a hot wire anemometer traversed across the porous surface of the model before the test.

Thin platinum film resistance thermometers, applied to Pyrex disk substrate and cemented with the film surface flush with the porous surface, provided the heat-transfer instrumentation. The output signal from the resistance thermometer reflecting the surface temperature history was transformed by analog circuits to the heat-transfer rate and observed directly on oscilloscopes. The precision of the measurements is a function of the accuracy of the thin-film gage calibration and the precision with which the signals can be measured on the oscilloscope traces. From these considerations, an accuracy between 10 and 15 per cent is expected. Since the effects of blowing should be much larger, this accuracy is quite adequate.

In addition to the heating rates, the tunnel flow parameters are obtained from a measurement of the reflected shock pressure and Mach number in the shock tube and the pitot pressure in the tunnel. These data are plotted in Figs. 1

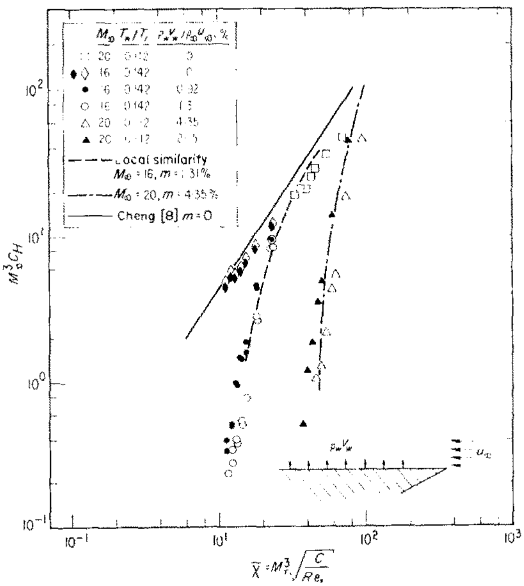

FIG. 1. Flat plate heat-transfer rate with mass injection, viscous interaction.

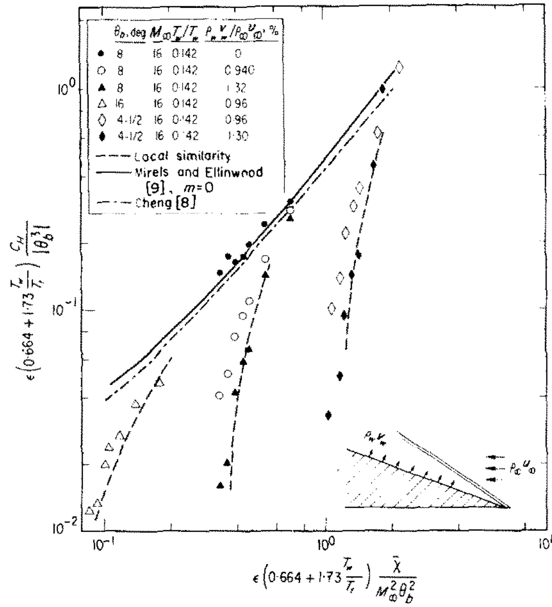

FIG. 2. Heat transfer with mass injection, viscous interaction on wedge, half-angle θ_b.

and 2 in the form of Stanton number $C_H = Q/\rho_\infty u_\infty(H_t - H_w)$ as a function of $\bar{\chi} = M_\infty^3\sqrt{(C/Re_x)}$ for the case of a flat plate at zero angle of attack (Fig. 1) and $C_H/|\theta_b^3|$ vs. $\bar{\chi}/M_\infty^2\theta_b^2$ for the case of a wedge (Fig. 2) with half angle θ_b (or plate at angle of attack θ_b). Included in the two graphs are Cheng's [8] viscous interaction predictions for no blowing for a flat plate and Mirels and Ellinwood's [9] predictions for a wedge of semivertex angle θ_b.

4. RESULTS AND DISCUSSION

In general, the experimental and analytical results, given in Figs. 1 and 2, are in good agreement. For the wedge, the best agreement occurs when the interaction is strong and the amount of blowing is small.

The results show that mass injection decreases the heat transfer rate below the nonblowing value. For a fixed value of m, these results show that the effects of mass injection decrease as the hypersonic viscous interaction. increases. This conclusion was anticipated in a previous discussion of the flo' parameters. This trend implies that with increasing viscous interaction the effect of pressure on heat transfer tends to override the effect of blowing. The consequence of this on slender, sharp nose, reentry vehicles is to minimize the effect of blowing near the nose as compared with that toward the aft end. In order for blowing to have an 0(1) effect on the boundary layer, it is required that $m \sim 0(\bar{\chi}^{\frac{1}{2}}/M_\infty^3)$ for flat plates and that $m \sim (\lambda^{\frac{1}{2}}K_b^2)/M_\infty^3$ for wedges.

REFERENCES

1. H. W. EMMONS and D. C. LEIGH, Tabulation of the Blasius function with blowing and suction, Fluid Motion Subcommittee Aeronautical Research Council F.M. 1913 (June 1953).
2. C. F. DEWEY, JR. and J. F. GROSS, Exact similar solutions of the laminar boundary layer equations, The Rand Corp., Santa Monica, Calif. (1963).
3. J. AROESTY and J. D. COLE, Boundary layer flows with large injection rates, Memorandum RM 4620-ARPA, The Rand Corp., Santa Monica, Calif. (August 1965).
4. T. KUBOTA and F. L. FERNANDEZ, Boundary layer flows with large injection and heat transfer, *AIAA Jl* **6**, 22–29 (1968).
5. W. S. KING, Integral solution for boundary layers with large blowing, TR-0200(4240-10)-5, The Aerospace Corp., El Segundo, Calif. (September 1968).
6. D. CATHERAL, K. STEWARTSON and P. G. WILLIAMS, Viscous flow past a flat plate with uniform injection, *Proc. R. Soc. (London)* **284A** (1398), 370–396 (March 1965).
7. W. D. HAYES and R. F. PROBSTEIN, *Hypersonic Flow Theory*, pp. 333–367. Academic Press, New York (1959).
8. H. K. CHENG, J. G. HALL, T. C. GOLIAN and A. HERTZBERG, Boundary layer displacement and leading edge bluntness effects in high temperature hypersonic flow, *J. Aerospace Sci.* **28**, 353–381 (1961).
9. H. MIRELS and J. W. ELLINWOOD, Hypersonic viscous interaction theory for slender axisymmetric bodies, TR-0158(3240-10)-1, The Aerospace Corp., El Segundo, Calif. (September 1967).
10. TING YI LI and J. F. GROSS, Hypersonic strong viscous interaction on a flat plate with surface mass transfer, *Heat Transfer and Fluid Mechanics Institute Proc.* 146 (June 1961).
11. W. S. KING, Hypersonic strong interaction on flat plate with uniform blowing, *AIAA Jl* **7**, 146–148 (January (1969).
12. C. F. DEWEY, JR., Use of local similarity concepts in hypersonic viscous interaction problems, *AIAA Jl* **1**, 20–33 (January 1963).
13. G. R. INGER, Laminar boundary layer solutions with strong blowing, *AIAA Jl* **5**, 1677–1678 (1967).

COUCHE LIMITE HYPERSONIQUE SUR UN DIÈDRE AVEC UNE ADDITION UNIFORME DE MASSE ET UNE INTERACTION VISQUEUSE.

Résumé—On étudie analytiquement et expérimentalement les effets de soufflage superficiel uniforme sur une couche limite hypersonique avec une interaction visqueuse.

Pour une interaction visqueuse forte ou modérée, le transfert thermique sur une plaque plane et sur un coin effilé est calculé en utilisant la technique de similitude locale pour résoudre les équations de couche limite et l'approximation du dièdre tangent pour déterminer la pression lorsqu'il n'y a pas viscosité. Des expériences sont réalisées à des nombres de Mach de 16 et 20, et des nombres unitaires de Reynolds de 2,3 et 1,3 . 10^5 ft^{-1}, et pour des conditions de paroi froide. Les résultats expérimentaux et théoriques sont en bon accord. Pour un soufflage modéré, on constate que les effets d'interaction visqueuse déterminent l'écoulement quand l'interaction est forte et que les effets de soufflage deviennent d'autant plus importants que l'intensité de l'interaction visqueuse décroît.

DIE HYPERSONISCHE GRENZSCHICHT AN EINEM KEIL MIT GLEICHMÄSSIGER MASSENZUFUHR UND ZÄHIGKEITSEINFLUSS

Zusammenfassung—Die Auswirkungen gleichmässiger Oberflächenausblasung auf die hypersonische Grenzschicht mit zähigkeitseinfluss werden analytisch und experimentell untersucht. Für starken und mässigen Zähigkeitseinfluss wird der Wärmetransport an einer ebenen Platte und einem schlanken Keil unter Benutzung der örtlichen Ähnlichkeit, um die Grenzschichtgleichungen zu lösen, und der "tangent-wedge"—Näherung berechnet, um den statischen Druck zu bestimmen. Die Experimente werden bei Mach-Zahlen von 16 und 20 und bei auf die Längeneiheit bezogenen Reynolds-Zahlen von 7,55 und 4,27 $\times$ 10^5 m^{-1} und bei kalter Wand ausgeführt. Die analytischen und experimentellen Ergebnisse stimmen gut überein. Für mässiges Ausblasen findet man, dass die Zähigkeitswirkungen überwiegen, wenn diese stark sind, und dass die Auswirkungen des Ausblasens entscheidend werden, sobald die Stärke des Zähigkeitseinflusses abnimmt.

СВЕРХЗВУКОВОЙ ПОГРАНИЧНЫЙ СЛОЙ НА КЛИНЕ ПРИ ОДНОРОДНОМ ВДУВЕ МАССЫ И ВЯЗКОМ ВЗАИМОДЕЙСТВИИ

Аннотация—Влияние однородного поверхностного вдува на сверхзвуковой пограничный слой при вязком взаимодействии исследуется аналитически и экспериментально. При сильных и средних вязких взаимодействиях теплоперенос на плоской пластине и тонком клине рассчитывается методом локальной автомодельности, для того чтобы решить уравнения пограничного слоя, и с помощью аппроксимации тангенциального клина для определения невязкого давления. Эксперименты проводились при числах Маха от 16 до 20, числах Рейнольдса от 2.3 до 1.3 $\times$ 10^5 фут$^{-1}$ и холодной стенке. Результаты аналитические и экспериментальные хорошо согласуются. При умеренных вдувах найдено, что влияние вязкого взаимодействия доминирует, если взаимодействие сильное, а эффекты вдува становятся более значительными по мере ослабления вязкого взаимодействия.